Multitask Learning and Prediction of Baseline Driving Performance Measures

SAFETY RESEARCH USING SIMULATION

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The Title Page - Report Title

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16. Abstract
Driving performance measures (DPMs) are important indices for driving and personal safety in vehicle operation. The DPMs are collected under various controlled driving conditions to demonstrate different driving behaviors so that mitigating technology interventions can be studied and designed. However, significant costs are involved in the DPM acquisition, and there are a very limited number of controlled driving condition data. Thus, the modeling and prediction of the DPMs under unobserved driving conditions are critical, and many methods have been developed. However, existing literature in this area suffer a common limitation: The interactions among different DPMs are not fully considered (each DPM is modeled individually), although the existence of such interactions is widely reported. This paper proposes a novel DPM modeling and prediction method, i.e., multi-output convolutional Gaussian process (MCGP), that incorporates the interactions among different DPMs. The method features the modeling flexibility for different DPMs and the interpretable modeling structure for integrating the DPM interactions. The method is compared with three benchmark methods on the DPM data set under four different settings, and the results demonstrate the superiorities of the method. Discussions and interpretations of the results are also provided.

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Abstract

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1 Introduction

Driving performance measures (DPMs) such as standard deviation of lateral lane position (SDLP) and standard deviation of driving speed (SDS), are important indices for driving and personal safety in vehicle operation (Verster & Roth, 2011). DPMs are widely used in impaired driving detection by comparing performance with the baseline measures of general driving performance. The key to such impaired driving detection is the comprehensive understanding of the baseline driving performance under various driving conditions. In most experimental cases, the DPM data is collected in a driving simulation laboratory or in controlled on-road studies, under various controlled driving and driver conditions (e.g., speed limit, age, driving experience). However, the data collection process can be expensive and time-consuming, so it is practically impossible to collect the baseline driving performance measures under all possible driving conditions and driver characteristics. Thus, modeling and prediction of the relationship between DMPs and driving conditions are important for understanding the baseline performance with limited data.

There are many studies on the relationship between driving conditions and single/specific DPM under baseline driving conditions (Hu et al., 2005; Tillyer et al., 2012; Verster & Roth, 2011). The generalized linear model (GLM) (McCullagh, 2019) is the most popular statistical tool to analyze such relationships. The GLM is a generalization of the ordinary linear regression, which allows various types of distributions in the response (DPM). This notable feature makes GLM a very practical choice for many real-world data analyses, including DPM analysis. Nevertheless, the DPM/driving condition relationship reported in literature are heterogeneous and sometimes even contradictory (Hu et al., 2005; Tillyer et al., 2012). For example, some studies reported the driver's age and gender have significant impacts on the the driver's reaction time (Caird et al., 2008; Hancock et al., 2003; Haque & Washington, 2014), while other studies observed that driver’s age and gender are not significantly related to the driver's reaction time (Choudhary & Velaga, 2017). There are two reasons that result in the difficulty and inconsistency of DPM analysis using GLM. First, the assumption of linear relationship in GLM might not be appropriate for many DPMs (Green & Silverman, 1993). For example, the relationship between traffic flow and traffic crashes cannot be sufficiently represented by linear model (Golob & Recker, 2003). As a result, the GLM significantly restricts the model flexibility in modeling the relationship, which can result in poor performance and inappropriate conclusions for complex relationships. Second, the GLM considers each DPM independently and ignores the inherent interactions among various DPMs. For instance, the SDLP and SDS are reported to be highly related with each other (Verster & Roth, 2014), which means knowing the data of SDLP can be informative to understand the SDS, and vice versa. As a result, it is preferable to analyze various DPMs jointly to fully exploit their interaction information, rather than treat each DPM individually or independently. Unfortunately, few works handle the analysis of multiple DPMs simultaneously, and many opportunities for understanding the relationship between DPMs and driving conditions are missed due to this limitation.

To overcome the issues in GLM methods, alternative approaches were studied and developed. For example, non-parametric and non-linear models are proposed to facilitate high flexibility in modeling performance. Some famous models include Gaussian process (GP) model (Lawrence & Hyvärinen, 2005), B-spline model (Chang & Yadama, 2010), and functional principal component analysis model (Satija & Caers, 2015). These models solve the first issue of GLM, i.e., relax the assumption of linear input-output relationship, thus provide more flexibilities in representing various input-output relationships. Among these emerging non-parametric and non-linear models, the GP model is prominent due to its flexibility in representing the data and the elegant mathematical form for interpretations (Rasmussen, 2003). Unfortunately, although the GP model is popular and widely used in civil, manufacturing, and biomedical engineering, this emerging technique has seen little application in modeling and predicting the relationship between DPMs and driving conditions. More importantly, existing GP models also focus on only one dependent variable (DPM). As a result, the direct application of the GP model cannot solve the issue of individual modeling, i.e., the second issue in GLM.

In this report, we propose to extend the existing univariate GP models to facilitate the simultaneous modeling and prediction of multiple DPMs, which enjoys the flexibility of GP models and overcomes the limitation of modeling individual variables in existing methods. The foundation of our model is to allow each DMP to have its unique feature, at the same time facilitate the interactions among DMPs through shared features. To realize this idea, we use the convolution process (CP) to construct the GP for each DPM. The CP is an effective way to construct GP by convolving a Gaussian white noise process with a smoothing kernel (Alvarez & Lawrence, 2011). In our proposed framework, each DPM is a GP resulting from the summation of two CPs. The first CP has a unique Gaussian white noise process for each DPM to represent each DPM's unique feature, while the second CP is shared among all the DPMs to represent their interactions. In this framework, each DPM can interact with other DPMs through the shared CP, at the same time possess its uniqueness. The proposed GP structure is denoted as multi-output convolutional Gaussian process (MCGP), where each 'output' represents a DPM. As a result, the proposed MCGP has three significant contributions to the modeling and prediction of relationship between DPMs and driving conditions: i) it facilitates the flexible and non-linear modeling of each DMP; ii) it considers the interactions among different DMPs when conducting modeling and prediction; iii) a clear interpretation between the proposed model and data is presented. The performance of the proposed model is compared with GLM methods and the univariate GP model using driving simulation data. The results demonstrate the superiority of the proposed method.

The rest of the report is organized as follows. Section 2 introduces the methodology, which includes the background introduction of univariate Gaussian process, assumptions and clarifications, and the technical details of the proposed method. In Section 3, the proposed method is applied to the lab simulated DPM data, and its modeling and prediction performance are compared with three benchmark methods. Finally, Section 4 draws the conclusion remarks.

2 Methodology

2.1 Background of univariate Gaussian process

In this subsection, we introduce the methodology background of univariate Gaussian process and the convolution process. A univariate Gaussian process is a collection of random variables that have a joint Gaussian distribution. Without loss of generality, we denote a pair of the driving condition and the corresponding DPM as $(\boldsymbol{x}, y(\boldsymbol{x}))$, where the driving condition is denoted as $\boldsymbol{x}$ and the measured univariate DPM is denoted as $y(\boldsymbol{x})$. Please note the $\boldsymbol{x} \in R^q, q \geq 1$, and $y(\boldsymbol{x}) \in R$, which means the driving conditions can be high-dimensional but the DPM is only allowed to be one-dimensional (univariate). For example, the driving conditions can be driver age, gender, and the number of lanes ($q = 3$), and the DPM is the measured SDLP (one-dimension) under the specified values of these driving conditions. Suppose there are n pairs (data) of $(\boldsymbol{x}, y(\boldsymbol{x}))$, the relationship between $\boldsymbol{x}$ and $y(\boldsymbol{x})$ is formulated by univariate Gaussian process as follows:

$$
\begin{gathered}
f(\boldsymbol{x}) \sim GP\big(\mu(\boldsymbol{x}), \Sigma(\boldsymbol{x}, \boldsymbol{x}')\big) \\
\boldsymbol{f}(\boldsymbol{x}_1) \sim MVN\big(\boldsymbol{\mu}(\boldsymbol{x}_1), \boldsymbol{\Sigma}(\boldsymbol{x}_1, \boldsymbol{x}_1')\big) \\
y(\boldsymbol{x}_{1i}) = f(\boldsymbol{x}_{1i}) + \epsilon(\boldsymbol{x}_{1i}),\ i = 1, \cdots n
\end{gathered}
\tag{2.1}
$$

where $f(\boldsymbol{x})$ is a GP with mean function $\mu(\boldsymbol{x})$ and co-variance function $\Sigma(\boldsymbol{x}, \boldsymbol{x}')$, $\boldsymbol{f}(\boldsymbol{x}_1) = [f(\boldsymbol{x}_{11}), \cdots, f(\boldsymbol{x}_{1n})]^T$ is the realization vector of $f(\boldsymbol{x})$ at n different driving conditions $\boldsymbol{x}_1 = [\boldsymbol{x}_{11}, \cdots, \boldsymbol{x}_{1n}]^T$ and $\boldsymbol{f}(\boldsymbol{x}_1)$ follows multivariate Normal (MNV) distribution with mean vector

$\boldsymbol{\mu}(\boldsymbol{x}_1) = [\mu(\boldsymbol{x}_{11}), \cdots, \mu(\boldsymbol{x}_{1n})]^T$ and $n \times n$ positive definite co-variance matrix $\boldsymbol{\Sigma}(\boldsymbol{x}_1, \boldsymbol{x}_1')$, and $\epsilon(\boldsymbol{x}_{1i})$ is the noise term that is assumed to follow independent and identically distributed (i.i.d.) Normal distribution with mean 0 and variance σ^2. In Eq. 2.1, the $\mu(\boldsymbol{x})$ and $\Sigma(\boldsymbol{x}, \boldsymbol{x}')$ determines the relationship between $\boldsymbol{x}$ and $y(\boldsymbol{x})$. The mean function $\mu(\boldsymbol{x})$ is usually set as 0 in the Gaussian process context (Rasmussen, 2003), so the relationship between $\boldsymbol{x}$ and $y(\boldsymbol{x})$ is fully characterized by the co-variance function $\Sigma(\boldsymbol{x}, \boldsymbol{x}')$.

The convolution process is a flexible way to construct the co-variance function $\Sigma(\boldsymbol{x}, \boldsymbol{x}')$ for a Gaussian process, which is based on the idea that a GP $f(\boldsymbol{x})$ can be constructed by convolving a Gaussian white noise process $\mathcal{W}(\boldsymbol{x})$ with a smoothing kernel $k(\boldsymbol{x})$:

$$f(\boldsymbol{x}) = k(\boldsymbol{x}) * \mathcal{W}(\boldsymbol{x}) = \int_{-\infty}^{+\infty} k(\boldsymbol{x} - \boldsymbol{u})\mathcal{W}(\boldsymbol{u})d\boldsymbol{u} \tag{2.2}$$

where $*$ is the convolution operator. The resulting co-variance can be formulated as:

$$\Sigma(\boldsymbol{x}, \boldsymbol{x}') = \int_{-\infty}^{+\infty} k(\boldsymbol{x} - \boldsymbol{u})k(\boldsymbol{x}' - \boldsymbol{u})d\boldsymbol{u} \tag{2.3}$$

where $\boldsymbol{x}$ and $\boldsymbol{x}'$ are two arbitrary driving condition values. If $\boldsymbol{x} = \boldsymbol{x}'$, then Eq. 2.3 represents the variance of $f(\boldsymbol{x})$, otherwise it denotes the co-variance between $f(\boldsymbol{x})$ and $f(\boldsymbol{x}')$. It is clear that the smoothing kernel $k(\boldsymbol{x})$ dominates the co-variance matrix, thus it determines the relationship between $\boldsymbol{x}$ and $y(\boldsymbol{x})$. In practice, the Gaussian kernel is usually used.

Equations 2.1 to 2.3 are the basic ideas of using the convolutional Gaussian process to model the relationship between driving conditions and univariate DPM. In this report, we extend it to model the interactions among multiple DPMs.

2.2 <u>Assumptions and clarifications</u>

To facilitate the modeling and prediction of the interactions among multiple DPMs using CP-based Gaussian process, we list some assumptions and clarifications as follows:

A1. The underlying relationship or function between driving conditions and each DPM is smooth. This assumption is made to satisfy the requirement of the Gaussian smoothing kernel.

A2. Different DPMs have the shared commonalities. In the context of Gaussian process, the 'commonalities' are depicted as the length-scale of DPM. This assumption is presented to make sure the modeling and prediction performance can be improved by capturing the interactions among different DPMs.

A3. Different DPMs share the same driving condition input space. For example, if the SDLP is measured under a certain number of driving conditions, e.g., driver age, driving experience, speed limit, then the SDS, if modeled together with SDLP, should also be collected under the same driving condition dimensions.

C1. The proposed method allows different number of observations in each DPM given the same input space.

We want to point out that the assumption A1 is a commonly used assumption for Gaussian process, and the assumption A2 lays the foundation for modeling the interactions among different DPMs. In practice, analysists can inquire transportation experts to get domain knowledge about which DPMs are potentially similar. If the domain knowledge is not available, practitioners can fit different DPMs two times: joint modeling (A2 is satisfied) and individual modeling (A2 is not satisfied). Then the generalized likelihood ratio test can be used to judge whether the joint modeling or the individual modeling is preferred. The assumption A3 is also straightforward since different DMPs in one driving simulation are typically collected from the same driver, which naturally satisfies the A3. Note that A3 requires the dimensions (categories) of driving conditions, rather than the driving condition settings, are consistent across different driving simulations. As a result, A3 allows different driving simulations to have different driving conditions and/or drivers. The clarification C1 adds the flexibility for the proposed method since it allows different number of observations for each DPM. This flexibility is specifically important when there are missing values in the recorded DPMs. For example, the SDLP and SDS are both collected under the same number of driving conditions (satisfying A3), but the recorded numbers of observations in SDLP and SDS are different. Such situation might happen when the required information for obtaining different DPMs is different, which causes the missed recordings for different DMPs. In these situations, given the same input driving conditions, the obtained number of observations can be different for different DPMs, and our proposed model is capable of handling such records.

2.3 Proposed method

In this subsection, we introduce the proposed method for modeling and prediction of the interactions among multiple DPMs. Without loss of generality, we assume there are l different DPMs and the j th DPM has n_j pairs data, which is denoted as $\boldsymbol{x}_j = \left[\boldsymbol{x}_{j1}, \cdots, \boldsymbol{x}_{jn_j}\right]^T$, $\boldsymbol{y}_j = \left[y(\boldsymbol{x}_{j1}), \cdots, y\left(\boldsymbol{x}_{jn_j}\right)\right]^T$, $j = 1, \cdots l$. Accordingly, the GP for each $\boldsymbol{y}_j$ is denoted as $f_j(\boldsymbol{x})$. The key to modeling the interactions among $\boldsymbol{y}_1, \cdots, \boldsymbol{y}_l$ is the way to construct the $f_j(\boldsymbol{x})$. As mentioned in the Introduction, the idea to construct $f_j(\boldsymbol{x})$ is to allow the unique feature in the jth DPM, at the same time depict the interactions among all the DPMs. As a result, our proposed structure for $f_j(\boldsymbol{x})$ is as follows:

$$f_j(\boldsymbol{x}) = k_{jj}(\boldsymbol{x}) * W_j(\boldsymbol{x}) + k_{0j}(\boldsymbol{x}) * W_0(\boldsymbol{x}) \tag{2.4}$$

where the $W_j(\boldsymbol{x})$ is the Gaussian white noise process that characterizes the unique feature of the jth DPM, the $W_0(\boldsymbol{x})$ is the shared Gaussian white noise process among all the DPMs, and the $k_{jj}(\boldsymbol{x})$ and $k_{0j}(\boldsymbol{x})$ are the kernels convolved with the corresponding Gaussian white noise process to construct the Gaussian process. In this model structure, the kernel $k_{0j}(\boldsymbol{x})$ can explicitly characterize the interactions between the jth DMP and other DMPs at any input x. By reading the kernel values, the interactions among DMPs can be quantified and interpreted. In this report, the Gaussian smoothing kernel will be adopted due to its flexibility to model different function features (Rasmussen, 2003).

Equation 2.4 reveals the mathematical interpretations for the interactions among l different DPMs. For example, two driving condition values $\boldsymbol{x}_{j,m}$ and $\boldsymbol{x}_{j',n}$, $j,j' = 1,\cdots,l$; $m = 1,\cdots,n_j$; $n = 1,\cdots,n_{j'}$, can be plugged into the Eq. 2.4, then the co-variance between the corresponding values in the jth DPM and the j'th DPM is:

$$Cov\left(f_j(\boldsymbol{x}_{j,m}), f_{j'}(\boldsymbol{x}_{j',n})\right) = \tau_{jj'} \int_{-\infty}^{+\infty} k_{jj}(\boldsymbol{x}_{j,m} - \boldsymbol{u}) k_{j'j'}(\boldsymbol{x}_{j',n} - \boldsymbol{u}) d\boldsymbol{u} + \int_{-\infty}^{+\infty} k_{0j}(\boldsymbol{x}_{j,m} - \boldsymbol{u}) k_{0j'}(\boldsymbol{x}_{j',n} - \boldsymbol{u}) d\boldsymbol{u} \quad (2.5)$$

where τ is a Kronecker delta function. We denote all the parameters in Eq. 2.5 as $\boldsymbol{\theta}$. In Eq. 2.5, if $j \neq j'$, it represents the co-variance (interaction) between the jth and j'th DPMs' observation values evaluated at $\boldsymbol{x}_{j,m}$ and $\boldsymbol{x}_{j',n}$, while the $j = j'$ depicts the co-variance (interaction) between two observation values in the same DPM (same as Eq. 2.3). If we plug vectors of driving conditions, i.e., $\boldsymbol{x}_j$ and $\boldsymbol{x}_{j'}$, into Eq. 2.5, then it becomes the co-variance matrix that represents the interactions between the jth DPM and the j'th DPM:

$$\Sigma_{j,j'} = Cov\left(\boldsymbol{f}_j(\boldsymbol{x}_j), \boldsymbol{f}_{j'}(\boldsymbol{x}_{j'})\right) = \begin{bmatrix} Cov\left(\boldsymbol{f}_j(\boldsymbol{x}_{j,1}), \boldsymbol{f}_{j'}(\boldsymbol{x}_{j',1})\right) & \cdots & Cov\left(\boldsymbol{f}_j(\boldsymbol{x}_{j,1}), \boldsymbol{f}_{j'}(\boldsymbol{x}_{j',n_{j'}})\right) \\ \vdots & \ddots & \vdots \\ Cov\left(\boldsymbol{f}_j(\boldsymbol{x}_{j,n_j}), \boldsymbol{f}_{j'}(\boldsymbol{x}_{j',1})\right) & \cdots & Cov\left(\boldsymbol{f}_j(\boldsymbol{x}_{j,n_j}), \boldsymbol{f}_{j'}(\boldsymbol{x}_{j',n_{j'}})\right) \end{bmatrix} \quad (2.6)$$

where we use the $\Sigma_{j,j'}$ to represent the co-variance matrix between the jth DPM and the j'th DPM. As a result, the interactions among all l DPMs can be represented as:

$$\Omega = \begin{bmatrix} \Sigma_{1,1} & \cdots & \Sigma_{1,l} \\ \vdots & \ddots & \vdots \\ \Sigma_{l,1} & \cdots & \Sigma_{l,l} \end{bmatrix} \quad (2.7)$$

The Eq. 2.7 provides a comprehensive view for modeling the interactions among l DPMs, where the diagonal block co-variance matrix $\Sigma_{j,j}, j = 1,\cdots,l$, describes the relationship among n_j data in the jth DPM, and the non-diagonal block co-variance matrix provide interaction information among different DPMs.

Finally, the prediction for an arbitrary driving condition value at any DPM, i.e., $\boldsymbol{x}_{j,i}^{*}$, can be obtained through some algebra, which follows a Normal distribution:

$$\begin{aligned} y_j\left(\boldsymbol{x}_{j,i}^{*}\right)|\boldsymbol{y} &\sim N\left(\mu^{*}, \sigma^{*2}\right) \\ \mu^{*} &= \Omega^{*}(\Omega + \boldsymbol{I}_N \sigma^2)^{-1}\boldsymbol{y} \\ \sigma^{*2} &= Cov\left(f_j\left(\boldsymbol{x}_{j,i}^{*}\right), f_{j'}\left(\boldsymbol{x}_{j,i}^{*}\right)\right) - \Omega^{*}(\Omega + \boldsymbol{I}_N \sigma^2)^{-1}\Omega^{*T} + \sigma^2 \end{aligned} \quad (2.8)$$

where $\boldsymbol{y} = \left[\boldsymbol{y}_1^T, \cdots, \boldsymbol{y}_l^T\right]^T$ represents all the available data in l DPMs, $\Omega^{*} = Cov\left(f_j\left(\boldsymbol{x}_{j,i}^{*}\right), \boldsymbol{y}\right)$, and $\boldsymbol{I}_N$ is the $N \times N$ identity matrix with $N = \sum_{j=1}^{l} n_j$. The μ^{*} is the prediction mean, which represents the predicted value for the jth DPM located at driving condition value $\boldsymbol{x}_{j,i}^{*}$. The σ^{*2} is the variance of the prediction, which represents the uncertainties/belief on the predicted DPM value. All the parameters in Eq. 2.8, including $\boldsymbol{\theta}$ and σ, can be estimated using the maximum likelihood estimation method (Ying, 1991).

A schematic illustration of the construction structure of the proposed MCGP is shown in Fig. 2.1, where we use only two-dimension driving conditions, i.e., driver age and driving miles per year ($q = 2$), and two DPMs, i.e., SDLP and SDS ($l = 2$), for the purpose of clear visualization. We choose the SDLP and SDS as the example because these two DPMs have strong correlations (Verster & Roth, 2014). The proposed framework can be easily extended to cases $q > 2$ and $l > 2$. In Fig. 2.1 (a), a two-output convolutional Gaussian

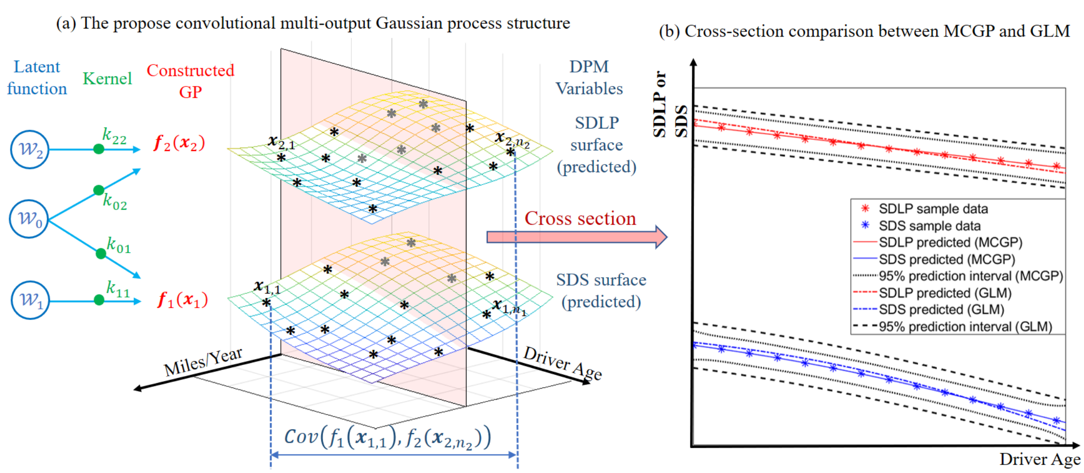

Figure 2.1 Illustration of the MCGP framework

process is constructed, where the first GP $f_1(\boldsymbol{x}_1)$ has its own unique $\mathcal{W}_1(\boldsymbol{x})$ and $k_{11}(\boldsymbol{x})$. The $f_1(\boldsymbol{x}_1)$ also shares a common $W_0(\boldsymbol{x})$ with $f_2(\boldsymbol{x}_2)$. This is a visualization of Eq. 2.4. As a result, the interactions between $f_1(\boldsymbol{x}_1)$ and $f_2(\boldsymbol{x}_2)$ are parametrized by $k_{01}(\boldsymbol{x})$ and $k_{02}(\boldsymbol{x})$, while the $k_{11}(\boldsymbol{x})$ and $k_{22}(\boldsymbol{x})$ characterize the unique features in $f_1(\boldsymbol{x}_1)$ and $f_2(\boldsymbol{x}_2)$, respectively. The black asterisks in Fig. 2.1 (a) are the measured data (SDLP and SDS), i.e., $y(\boldsymbol{x})$, under specific driving conditions, i.e., $\boldsymbol{x}$. Based on these data and using Eq. 2.8, the mean values of DPMs at un-measured driving conditions can be predicted, and the predicted values for two DPMs are represented as the two surfaces in Fig. 2.1 (a). To demonstrate the prediction accuracy and the quantification of the uncertainties, Fig. 2.1 (b) demonstrates the prediction results on the cross-section at a given driving condition value. Both the prediction means and uncertainties (95% intervals) are presented and compared with the results from the GLM method. Please note the GLM method fit each DPM individually, thus does not consider the interactions between SDLP and SDS. It is clear that the proposed MCGP provides more accurate prediction and smaller uncertainties. The more accurate prediction results demonstrate the flexibility of the GP, which can adapt to the various relationship between driving condition and DPM. The smaller uncertainties manifest the higher ‘confidence’ about the predictions the MCGP presents. The better performance of MCGP in Fig. 2.1 (b) is the direct result and evidence that validate the flexibility and superiority of the proposed structure in Fig. 2.1 (a). We will demonstrate more comprehensive comparisons in the next section.

3 Performance Comparison

3.1 Data illustration

Data used in this report was collected prior to 2019 from driving simulator research studies conducted at the National Advanced Driving Simulator (NADS) on the NADS-1 and miniSim simulator platforms for normal vehicle operation. The NADS-1, owned by

National Highway Traffic Safety Administration (NHTSA) and operated by the University of Iowa, is one of the highest-fidelity simulators in the world (see Fig. 3.1) with 13-degrees of freedom (dof), 360-degree horizontal visuals, and surround sound providing drivers with the most immersive environment possible. The NADS miniSim is a quarter cab "open buck" simulator (see Fig. 3.2) providing a more limited horizontal field of view and surround sound but no motion cueing.

Figure 3.1 NADS-1 dome—exterior (left), interior (right)

Figure 3.2 miniSim driving simulator with a quarter cab

The collected data is ‘baseline’ data, which refers to the conditions devoid of driving impairment such alcohol, drugs, drowsiness, and external distraction. Experimental drives were designed to address the research aims of the individual studies and we examined the general driving performance in areas where there were no experimental events (e.g., prior to a lead vehicle braking event). Driving environments were built to align with appropriate Federal Highway Administration Road Design Guidelines, and the Manual on Uniform Traffic Control Devices (MUTCD). All data was collected as part of research studies approved by the University of Iowa with approval Institutional Review Boards (FWA000003007). Participants in the studies provided inform consent prior to participation. Participant handling procedures differed by study but in general, all subjects were provided an overview of the research activities that would occur and had at least one opportunity to practice driving the simulator before the driving performance data under consideration in this analysis were collected. Data from subjects who were impaired is not included in this analysis. The general process was for subjects to provide informed consent prior being completing demographic and other surveys, training on study procedures, and completing a practice drive on the simulator. Subjects were instructed to drive as they normally would through the driving environments presented (e.g., urban, rural, interstate). Driver inputs and vehicle control measures were collected at between 60 and 240 Hz across the drive and reduced to analyzable measures such as SDLP, SDS, steering reversal rate, and lane departures. Readers interested in more details about the data repository can refer to (Brown et al., 2019).

Extremely large streams of data were recorded and collected in this study, which include driver’s demographic data (age, sex, race, ethnicity, miles driven per year, years of driving experience), driving environment data (speed limit, presence of traffic, number of lanes, event duration, time of the day, pavement condition) and simulator data (standard deviation of lateral lane position (SDLP), standard deviation of driving speed (SDS), simulator platform, number of lane departures left, number of lane departures right, standard deviation of steer, steer speed, steer angle, steer reversal rate). Pre-processing of the data was conducted to handle missing values and incorrect values, histograms of data was plotted to identify the outliers visually, also outliers with a Cook's distance greater than 0.5 were identified and removed, and imbalanced multiple records at each input are re-balanced by using their weighted average. After the pre-processing, the data become organized and ready for further analysis, and the three driving performance measures (SDLP, SDS and Standard deviation of steer) were initially selected as the dependent variables for our pre-analysis. As the focus of this report is to demonstrate and validate that our proposed model MCGPM is capable of capturing and exploiting the correlation among different DPMs to improve the prediction performance, we need to locate at least two DPMs that are strongly correlated. However, Standard deviation of steer did not demonstrate strong correlation with SDLP and SDS in our pre-analysis. Thus, we choose to use the SDLP and SDS as the final objects to conduct our analysis. Generalized linear models (GLM) were used to describe the relationships between independent and dependent variables. Akaike information criterion (AIC) (Sakamoto et al., 1986) and Bayesian information criterion (BIC) (Schwarz, 1978) were applied on this GLM models to identify the significant contributors (by removing redundant co-variates and collinearity). After these procedures, we identify significant independent variables as Age, Speed limit, Presence of traffic, Number of lanes and Simulator platform, and the two DPMs (SDLP and SDS) which are strongly correlated. In this report, we focus on these five driving conditions and two DPMs. These driving conditions and DPMs are also widely reported in literature to be effective in demonstrating the driving behaviors (Antin et al., 2020; Guériau et al., 2020; Haglund & Åberg, 2000; Sawalha & Sayed, 2001; Verster & Roth, 2011). For example, the SDLP is widely used in evaluating the ability to track road, to remain within the lane, and to maintain constant speed. The SDS is a common measure for assessing driving performance in drugged driving research. We define the driving conditions and DPMs as 'independent variables' and 'dependent variables', respectively. These variables are introduced in detailed as follows:

Independent variables: Speed limit, Presence of traffic and Number of lanes are the three road characteristics that reflect observed differences in driving environment. The driver age is a measure of driver characteristics. Additionally, the simulator platform on which the data was collected is also included. These independent variables are summarized in Table 1. Please note that it was not possible to examine all combinations of independent variable (e.g., low speed limits don't exist on high speed interstates and high speeds don't exist in low speed residential/urban areas).

Table 3.1 Independent Variables

Variable	Description	Range	Units
Speed Limit	The posted speed limit (in mph) of the roadway segment.	10 to 70	Miles per hour (mph)
Presence of Traffic	Whether oncoming (for two-lane roads) or adjacent traffic (for multi-lane divided highways) was present in the roadway segment.	Yes or No	-
Number of Lanes	The number of lanes in the driving direction.	1 or 2	-
Driver Age	The age at the time of enrollment of the participant based on birthdate and, if not present, self-reported age.	16 to 89	Years
Simulator Platform	Either the NADS-1 or the miniSim.	NADS-1 or miniSim	-

Dependent variables: Two dependent measures, i.e., SDLP and SDS, are selected for analysis. The SDLP is widely used to assess the quality of lateral vehicle control, particularly in impaired driving research, by evaluating the ability of the driver to track the road and maintain position in the lane. The SDS is a parallel measure to SDLP for longitudinal control which assesses the quality of speed control. These two measures are documented in Table 2.

Table 3.2 Dependent Variables

Variable	Description	Units
Standard Deviation of lateral Lane Position (SDLP)	A measure of the stability of the lateral control of the vehicle based on variability around the mean lane position for a segment.	Centimeters
Standard Deviation of driving Speed (SDS)	A measure of longitudinal control of the vehicle reflecting variability in speed maintenance within a segment.	Meters per second (m/s)

To facilitate the visualization and evaluation of the proposed method, we only demonstrate the influence of the 'driver age' and 'speed limit' on the DPMs. The 'presence of traffic', 'number of lanes', and 'simulation platform' are used as subset indices to differentiate the influence of the 'driver age' and 'speed limit' on the DPMs under different scenarios. To make a fair comparison among different subsets and present the results concisely, we select 4 out of 8 subsets to demonstrate the model performance. The subset settings are shown in Table 3. We select these subsets based on the following criterion: 1) The number of data points in each subset should be close to each other so that the prediction accuracy is comparable among subsets; 2) The NADS-1 provides more accurate measures than the miniSim, so we choose more subsets conducted on NADS-1. Although the miniSim is less comprehensive than NADS-1, we still include it because it is easy to setup and its operation costs are cheaper than NADS-1. This makes it widely used for data collection process (Kay et al., 2013; Lee et al., 2013; Winton et al., 2015).

Therefore, to demonstrate that our method can work on different data collection platform, we include the miniSim and NADS-1 simultaneously and pay more attention to the NADS-1; 3) The interactions among DMPs can be more complicated when there presents the traffic, thus we choose more subsets that present the traffic to clearly show the interactions among DMPs.

Moreover, the data used in our final analysis went through the pre-analysis step because of which these four subsets did not have uniform distribution of the data. Fortunately, our proposed method is based on the Gaussian process, which is widely known for allowing random selection of input data points. This means our method can handle the random missing data from the pre-analysis, this feature is another unique advantage of our proposed method which confirms its superiority over the benchmark methods.

Table 3.3 Subsets settings for model evaluations

	Number of Lanes	Presence of Traffic	Simulator Platform	Inputs Variables	Outputs Variables
Subset 1	1	Yes	NADS-1	Age, Speed limit	SDLP, SDS
Subset 2	1	Yes	miniSim	Age, Speed limit	SDLP, SDS
Subset 3	2	Yes	NADS-1	Age, Speed limit	SDLP, SDS
Subset 4	1	No	NADS-1	Age, Speed limit	SDLP, SDS

The data with 426 observations are split into training data and testing data. In this report, we randomly choose 80% of the data as the training data, and the rest 20% as testing data. This procedure is conducted for each subset. As a result, all the models will use the training data to train or estimate the parameters, then the prediction results are evaluated at the testing data locations. The Root Mean Squared Error (RMSE) between the predictions and the testing data values is used as the evaluation criterion. To avoid over-fitting and training/testing data selection bias, cross-validation is applied, and the results in each subset are repeated 25 times.

3.2 Benchmark methods

The three benchmark methods applied and compared in this report were strategically selected. They are listed and elaborated below.

1. Generalized linear model: The generalized linear model (McCullagh, 2019) is set as the most standard benchmark. It includes all the five drive condition variables and fits the relationship between these five variables and the two DPMs. Please note there is only one fitting result in this benchmark, and this fitted model can be used for each of the 4 subsets (by assigning the corresponding values of the 'presence of traffic', 'number of lanes', and 'simulation platform' to the inputs variables).

- *Inputs*: Speed limit, Presence of traffic, Number of lanes, Driver age, and Simulator platform
- *Outputs*: SDLP and SDS (independently)

2. Specific generalized linear model: To improve the performance of the overall GLM, the GLM is fitted in each subset, which we call the specific generalized linear model (SLM). In this benchmark, the GLM is applied to each subset data, and the inputs become speed limit and driver age. Four different SLM are obtained, and their performances are evaluated and compared. The major difference between the GLM and the SLM is that GLM fits all the subsets in one time, while the SLM fits each subset specifically. It can be expected that the GLM is more time efficient

than SLM, but the SLM adds flexibility (regression degree of freedom) to the data regression.

- *Inputs*: Speed limit, and Driver age
- *Outputs*: SDLP and SDS (independently)

3. Univariate Gaussian process: Both the GLM and SLM are parametric and linear models, so to compare these models with non-parametric and non-linear models, univariate Gaussian process (Rasmussen, 2003) was applied, and this model is referred to as Gaussian process model (GPM). The GPM is applied to each of the four subsets (same input and output setting as SLM).

- *Inputs*: Speed limit, and Driver age
- *Outputs*: SDLP and SDS (independently)

3.3 Performance demonstration

It is clear all the three benchmark methods can only fit the two DPMs independently. So, our proposed method, which is referred as multi-output convoluted Gaussian process model (MCGPM), is applied to each of the four subsets to demonstrate the effectiveness of incorporating the interactions in DPMs.

- Inputs: Speed limit, and Driver age
- Outputs: SDLP and SDS (dependently)

We report the Boxplots for 25 trials in each subset for each method. The RMSE of SDLP and SDS for subsets 1-4 are reported in Figs. 4-7, respectively. To facilitate the visualization of the modeling and prediction performance, the fitting results, including the training and testing data, are also demonstrated in Figs. 4-7. Please note to keep the clearness of the visualization, we only demonstrated the visual results of GLM and the proposed method (MCGPM). Also note that the range and distribution in Figs. 4-7 might not exactly match that in Table 1, which is because the Table 1 provides information about all the data after pre-analysis, while the Figs. 4-7 only provide data in each sub-set. Readers interested in more detailed results can contact the authors to inquire more figures and codes for the results.

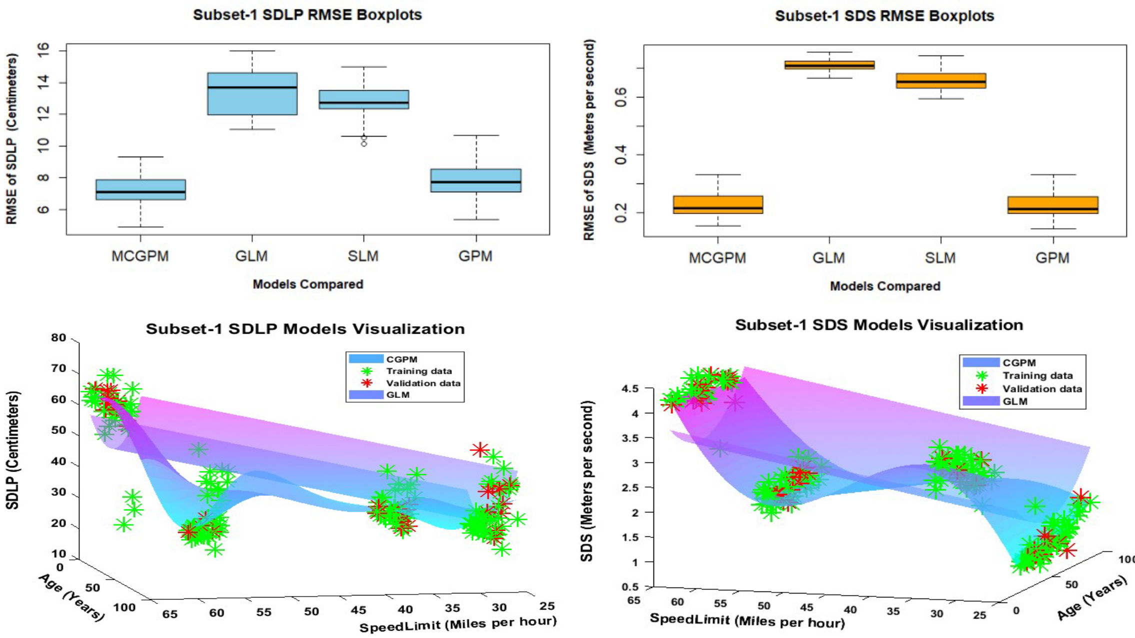

Figure 3.3 Subset 1 RMSE boxplots and models visualization

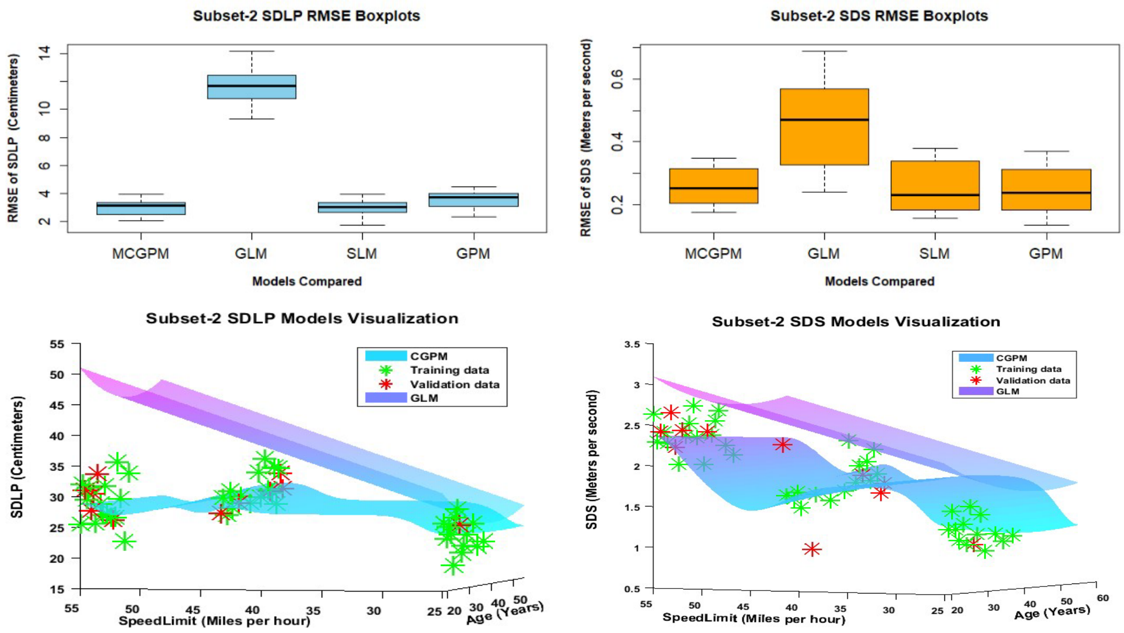

Figure 3.4 Subset 2 RMSE boxplots and models visualization

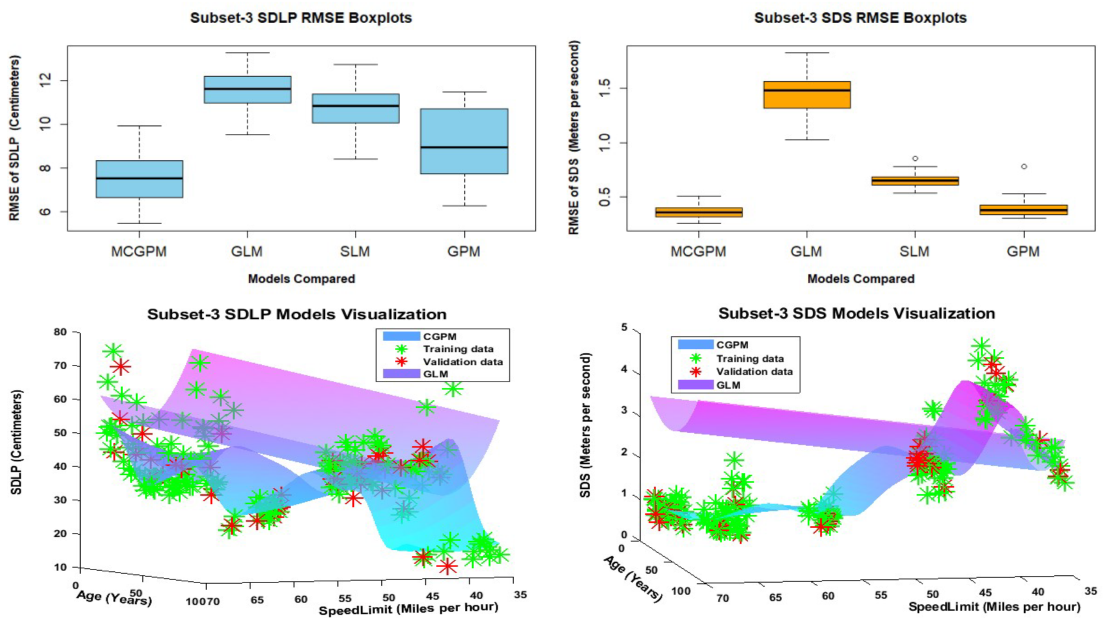

Figure 3.5 Subset 3 RMSE boxplots and models visualization

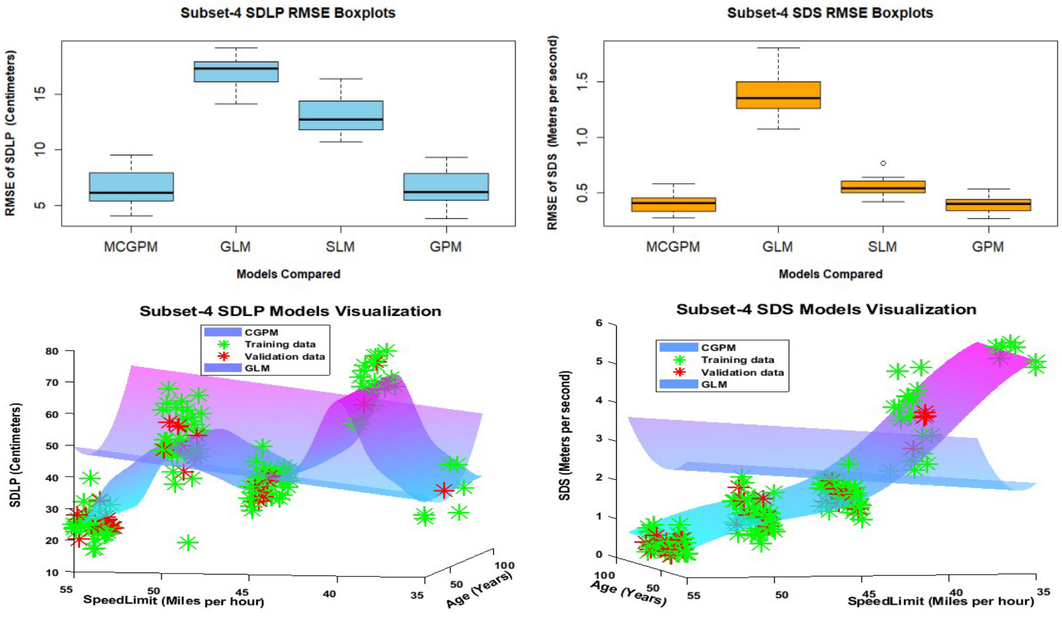

Figure 3.6 Subset 4 RMSE boxplots and models visualization

3.4 <u>Discussion</u>

The Figs. 3.3-3.6 clearly show the proposed method (MCGPM) performs the best consistently across different subsets. But the results demonstrated in these figures reveal more interesting insights and discoveries about the DPM behaviors and modeling features. We conclude the discoveries as follows:

D1. *One-time fitting vs. specific fitting*. In all the subsets, the GLM performs consistently the worst, and the SLM always outperforms the GLM. Recall that the only difference between the GLM and SLM is that the SLM fits all the subsets individually while the GLM fits all the subsets together. It is also obvious in the visualization graphs in Fig. 3.6 that the GLM even presents the wrong trend between Speed Limit and SDS. This is a direct result of the misleading 'downwards' trend in subsets 1 and 2, which influences the regression trend in the subset 4. This means the relationship between the driving conditions and DPMs are very complicated, and a simple GLM cannot handle such complicated relationship in a 'one-time' regression. This observation is consistent with the argument from Hu et al., (2005) and Tillyer et al., (2012) that the fitted relationships between the driving conditions and DPMs are heterogeneous and sometimes even contradictory. It also supports our decision in splitting the dataset into different subsets to better demonstrate and visualize the performance.

D2. *Parametric vs. Non-parametric*. In the Boxplots, the SLM sometimes performs very close to the non-parametric methods, e.g., Fig. 3.4, but it also encounters inferior performance in other cases, e.g., Fig. 3.3. The underlying reason for such unstable performance for the SLM is revealed by the visualization graphs. For example, in Fig. 3.3, the data or relationship between driving conditions and the DPMs are so complicated that the linear model cannot represent correctly. While in Fig. 3.4, the relationship becomes almost flat, which is suitable for linear models. Such observations are clearer in Fig. 3.6, where the relationship in SDLP is more complex than that in SDS. This makes the SLM perform better in SDS. As a result, the parametric works well only under specific cases, which limits their applications in modeling and predicting the complex relationship.

D3. *Dependent outputs vs. Independent outputs*. Although our performance comparison in Figs. 3.3-3.6 shows the MCGPM consistently outperforms the GPM, the specific model performance in each subset is different. For example, the MCGPM and GPM are almost the same in Fig. 3.6, while the MCGPM clearly outperforms the GPM in Figs. 3.3-3.5. By analyzing the visualization graphs, we realize this is a clear evidence that the performance of MCGPM depends on the interactions between the SDLP and SDS. In Figs. 3.3-3.5, the SDLP and SDS behave in a clearly similar way. For example, the SDLP and SDS are both flat in Fig. 3.4, and there is a common peak around Speed Limit=40 mph in SDLP and

SDS in Fig. 3.5. However, the SDLP and SDS in Fig. 3.6 behaves heterogeneously. Nevertheless, the different behaviors of SDLP and SDS in Fig. 3.6 do not make the MCGPM inferior to GPM. This means the MCGPM can model output interactions adaptively: if the MCGPM can capture the outputs' interactions, it can benefit from the dependent outputs. If the interactions in outputs cannot be captured by MCGPM, then the MCGPM behaves similar to the GPM.

D4. *Detection criteria of abnormal driving behaviors*. The studies conducted in this report is based on the baseline driving performance measures, but the impacts of our studies can go beyond the baseline performance. This is because the baseline driving performance measures serve as the important base for judging and deciding whether a recorded driving behavior is abnormal. The analysis results demonstrate that different subsets (driving conditions) give heterogeneous baseline DPM trends and relationships. This reveals the needs of establishing different detection criterion under different driving conditions. For example, the detection criterion for driving under one lane and two lanes should be different. Another insight for setting the detection criterion for abnormal driving behaviors is to include the interactions among DPMs. The rationale is that the modeling and prediction of DPMs under baseline condition can benefit from the incorporation of their interactions, so the inclusion of DPM interactions into abnormal driving detection can also deliver a more accurate and efficient detection.

4 Conclusions

In this report, the relationship between driving conditions and DPMs are studied. A novel multi-output convolutional Gaussian process model is proposed to model such relationship. In this model, each DPM is treated as an 'output' and constructed by two convolution (Gaussian) processes. The first convolution process is designed to represent the unique features in the DPM, and the second convolution process aims at capturing the shard features (interactions) among different DPMs. This structure is realized by the convolution process between white noise Gaussian process and Gaussian smoothing kernels. The uniqueness and contribution of the proposed model are that it facilitates the interactions among different DPMs. Comparing with existing models, the proposed model can take advantages of the interactions to improve the modeling and prediction performance. The performance of the proposed method is compared with two parametric linear models and the univariate Gaussian process model. The results demonstrate the proposed method is flexible in representing each DPM, at the same capable of capturing the interactions among DPMs.

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